Theory of microwave-induced oscillations in the magnetoconductivity of a 2D electron gas

I.A. Dmitriev[1,*], M.G. Vavilov[2], I.L. Aleiner[3], A.D. Mirlin[1,4,†], and D.G. Polyakov[1,*]
[1] Institut für Nanotechnologie, Forschungszentrum Karlsruhe, 76021 Karlsruhe, Germany
[2] Center for Materials Sciences and Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139, USA
[3] Physics Department, Columbia University, New York, NY 10027, USA
[4] Institut für Theorie der Kondensierten Materie, Universität Karlsruhe, 76128 Karlsruhe, Germany

We develop a theory of magnetooscillations in the photoconductivity of a two-dimensional electron gas observed in recent experiments. The effect is governed by a change of the electron distribution function induced by the microwave radiation. We analyze a nonlinearity with respect to both the *dc* field and the microwave power, as well as the temperature dependence determined by the inelastic relaxation rate.

PACS numbers: 73.40.-c, 78.67.-n, 73.43.-f, 76.40.+b

I. INTRODUCTION

Recent experiments have discovered[1] that the resistivity of a high-mobility two-dimensional electron gas (2DEG) in GaAs/AlGaAs heterostructures subjected to microwave radiation of frequency ω exhibits magnetooscillations governed by the ratio ω/ω_c, where ω_c is the cyclotron frequency. Subsequent work[2–7] has shown that for samples with a very high mobility and for high radiation power the minima of the oscillations evolve into zero-resistance states (ZRS).

These spectacular observations have attracted much theoretical interest. As was shown in Ref. 8, the ZRS can be understood as a direct consequence of the oscillatory photoconductivity (OPC), provided that the latter may become negative. A negative value of the OPC signifies an instability leading to the formation of spontaneous-current domains showing zero value of the observable resistance. Therefore, the identification of the microscopic mechanism of the OPC appears to be the key question in the interpretation of the data.[1–7]

A mechanism of the OPC proposed in Ref. 9 is based on the effect of microwave radiation on electron scattering by impurities in a strong magnetic field (see also Ref. 10 for an earlier theory and Ref. 11 for a systematic theory). An alternative mechanism of the OPC was recently proposed in Ref. 12. In contrast to Refs. 9–11, this mechanism is governed by a radiation-induced change of the electron distribution function. Because of the oscillations of the density of states (DOS), $\nu(\varepsilon)$, related to the Landau quantization, the correction to the distribution function acquires an oscillatory structure as well. This generates a contribution to the *dc* conductivity which oscillates with varying ω/ω_c. A distinctive feature of the contribution of Ref. 12 is that it is proportional to the inelastic relaxation time $\tau_{\rm in}$. A comparison of the results of Refs. 11 and 12 shows that the latter contribution dominates if $\tau_{\rm in} \gg \tau_{\rm q}$ (where $\tau_{\rm q}$ is the quantum, or single-particle, relaxation time due to impurity scattering), which is the case for the experimentally relevant temperatures.

The consideration of Ref. 12 is restricted to the regime which is linear in both the *ac* power and the *dc* electric field. The purpose of this paper is to develop a complete theory of the OPC governed by this mechanism, including nonlinear effects. We will demonstrate that the conductivity at a minimum becomes negative for a large microwave power and that a positive sign is restored for a strong *dc* bias, as it was assumed in Ref. 8.

The paper is organized as follows. First, in Sec. II we formulate a general approach to the problem. In Sec. III we calculate the non-equilibrium distribution function for overlapping Landau levels (LLs). In Sec. IV we consider the OPC in the linear regime with respect to the *dc* field. In Sec. V we analyze the ZRS and calculate the spontaneous electric field in the domains. In Sec. VI we turn to separated LLs. Section VII deals with the inelastic relaxation due to electron-electron scattering. Finally, in Sec. VIII we briefly discuss the magnetooscillations in the Hall photoresistivity. In Sec. IX we summarize our results and compare them with the experimental data. A brief account of the results of this paper was presented in Ref. 13.

II. GENERAL FORMALISM

We consider a 2DEG (mass m, density n_e, Fermi velocity v_F) subjected to a transverse magnetic field $B = (mc/e)\,\omega_c$. We assume that the field is classically strong, $\omega_c\tau_{\rm tr} \gg 1$, where $\tau_{\rm tr}$ is the transport relaxation time at $B = 0$. The photoconductivity $\sigma_{\rm ph}$ determines the longitudinal current flowing in response to a *dc* electric field $\mathcal{E}_{\rm dc}$, $\vec{j}\cdot\vec{\mathcal{E}}_{\rm dc} = \sigma_{\rm ph}\mathcal{E}_{\rm dc}^2$, in the presence of a microwave electric field $\mathcal{E}_\omega \cos\omega t$. The more frequently measured[1–4,6,7] longitudinal resistivity, $\rho_{\rm ph}$, is given by $\rho_{\rm ph} \simeq \rho_{xy}^2\sigma_{\rm ph}$, where $\rho_{xy} \simeq eB/n_e c$ is the Hall resistivity, affected only weakly by the radiation.

We start with the formula for the *dc* conductivity per spin:

$$\sigma_{\rm ph}=\int d\varepsilon\,\sigma_{\rm dc}(\varepsilon)\,\left[-\partial_\varepsilon f(\varepsilon)\right], \qquad (1)$$

where $f(\varepsilon)$ is the electron distribution function, and $\sigma_{\rm dc}(\varepsilon)$ determines the contribution of electrons with energy ε to the dissipative transport. In the leading approximation,[11,12]

$$\sigma_{\rm dc}(\varepsilon)=\frac{e^2\nu(\varepsilon)v_F^2}{2}\,\frac{\tau_{\rm tr,B}^{-1}(\varepsilon)}{\omega_c^2+\tau_{\rm tr,B}^{-2}(\varepsilon)}, \qquad (2)$$

where $\tau_{\rm tr,B}$ is the transport scattering time in a quantizing magnetic field, $\tau_{\rm tr,B}(\varepsilon)=\tau_{\rm tr}\nu_0/\nu(\varepsilon)$, and $\nu_0=m/2\pi$ is the DOS per spin at zero B (we use $\hbar=1$). We note that Eq. (2) has a Drude form with the DOS $\nu(\varepsilon)$ and the transport time $\tau_{\rm tr,B}(\varepsilon)$ dependent of energy due to the Landau quantization.

For a classically strong magnetic field, $\omega_c\tau_{\rm tr}\gg 1$, the above expression reduces to

$$\sigma_{\rm dc}(\varepsilon)=\sigma_{\rm dc}^{\rm D}\,\tilde\nu^2(\varepsilon), \qquad (3)$$

where $\sigma_{\rm dc}^{\rm D}=e^2\nu_0v_{\rm F}^2/2\omega_c^2\tau_{\rm tr}$ is the *dc* Drude conductivity per spin in strong B and we introduced the dimensionless DOS, $\tilde\nu(\varepsilon)=\nu(\varepsilon)/\nu_0$.

We neglect here the effect of the microwaves on the impurity collision integral, which yields a subleading contribution to the phoconductivity, as discussed in Sec. IV. The dominant effect is due to a non-trivial energy dependence of the non-equilibrium distribution function $f(\varepsilon)$. The latter is found as a solution of the stationary kinetic equation for the zero angular harmonic of the distribution function $f(\varepsilon)$:

$$\mathrm{St}_\omega\{f\}+\mathrm{St}_{\rm dc}\{f\}=-\mathrm{St}_{\rm in}\{f\}\,. \qquad (4)$$

Here the left–hand side represents the effect of the microwaves (St_ω) and of the *dc* field ($\mathrm{St}_{\rm dc}$) in the presence of impurities while the right–hand side accounts for the inelastic relaxation.

The first term on the left–hand side describes the absorption and emission of microwave quanta; the rate of these transitions was calculated in Ref. 12, yielding

$$\begin{aligned}\mathrm{St}_\omega\{f\}=\frac{\mathcal{E}_\omega^2}{4\omega^2}\sum_\pm\frac{e^2v_F^2\,\tau_{\rm tr,B}^{-1}(\varepsilon+\omega)\left[f(\varepsilon+\omega)-f(\varepsilon)\right]}{2(\omega\pm\omega_c)^2+\tau_{\rm tr,B}^{-2}(\varepsilon+\omega)+\tau_{\rm tr,B}^{-2}(\varepsilon)}\\ +\{\omega\to-\omega\}\,. \qquad (5)\end{aligned}$$

We will assume that $|\omega\pm\omega_c|\gg\tau_{\rm tr,B}^{-1}$, thus excluding a narrow vicinity of the cyclotron resonance. This allows us to neglect the ε–dependent terms in the denominator, which reduces Eq. (5) to the form

$$\mathrm{St}_\omega\{f\}=\mathcal{E}_\omega^2\,\frac{\sigma_\omega^{\rm D}}{2\omega^2\nu_0}\sum_\pm\tilde\nu(\varepsilon\pm\omega)\left[f(\varepsilon\pm\omega)-f(\varepsilon)\right], \qquad (6)$$

where the *ac* Drude conductivity per spin is given by

$$\sigma_\omega^{\rm D}=\sum_\pm\frac{e^2\nu_0v_{\rm F}^2}{4\tau_{\rm tr}(\omega\pm\omega_c)^2}. \qquad (7)$$

Furthermore, we assume, in accordance with the experiments, a linear polarization of the microwaves. For a circular polarization, one should retain only one term on the right-hand side of Eq. (7), which, away from the cyclotron resonance, does not affect the results in any essential way.

The second term on the left–hand side of Eq. (4) represents the effect of the *dc* electric field. The impurity scattering in a quantizing magnetic field leads to the spatial diffusion with a diffusion coefficient $D_{\rm B}(\varepsilon)=v_F^2/2\omega_c^2\tau_{\rm tr,B}(\varepsilon)$. In view of conservation of the total energy $\varepsilon+e\mathcal{E}_{\rm dc}x$ in a *dc* field (directed along the x axis) the spatial diffusion is translated into the diffusion in ε-space, with a diffusion coefficient $(e\mathcal{E}_{\rm dc})^2D_{\rm B}(\varepsilon)$ and the DOS $\nu(\varepsilon)$,

$$\begin{aligned}\mathrm{St}_{\rm dc}\{f\}&=\nu^{-1}(\varepsilon)\,\partial_\varepsilon\left[\,\nu(\varepsilon)\,e^2\mathcal{E}_{\rm dc}^2D_{\rm B}(\varepsilon)\,\partial_\varepsilon f(\varepsilon)\,\right]\\ &=\mathcal{E}_{\rm dc}^2\,\frac{\sigma_{\rm dc}^{\rm D}}{\nu_0\tilde\nu(\varepsilon)}\,\partial_\varepsilon\left[\,\tilde\nu^2(\varepsilon)\,\partial_\varepsilon f(\varepsilon)\,\right]. \qquad (8)\end{aligned}$$

Equation (8) can also be obtained from Eq. (6) by taking the limit $\omega\to 0$ and replacing the period-average of the *ac* field squared, $\frac{1}{2}\mathcal{E}_\omega^2$, by $\mathcal{E}_{\rm dc}^2$.

Substituting Eqs. (6),(8) in the kinetic equation (4), we get

$$\begin{aligned}&\mathcal{E}_\omega^2\,\frac{\sigma_\omega^{\rm D}}{2\omega^2\nu_0}\sum_\pm\tilde\nu(\varepsilon\pm\omega)\left[f(\varepsilon\pm\omega)-f(\varepsilon)\right]\\ &+\mathcal{E}_{\rm dc}^2\,\frac{\sigma_{\rm dc}^{\rm D}}{\nu_0\tilde\nu(\varepsilon)}\,\partial_\varepsilon\left[\,\tilde\nu^2(\varepsilon)\,\partial_\varepsilon f(\varepsilon)\,\right]=\frac{f(\varepsilon)-f_T(\varepsilon)}{\tau_{\rm in}}, \qquad (9)\end{aligned}$$

where the inelastic processes are included in the relaxation time approximation and $f_T(\varepsilon)$ is the Fermi distribution. A detailed discussion of the inelastic relaxation and a calculation of the inelastic relaxation time $\tau_{\rm in}$ are relegated to Sec. VII.

Equation (9) suggests convenient dimensionless units for the strength of the *ac* and *dc* fields:

$$\mathcal{P}_\omega=\frac{\tau_{\rm in}}{\tau_{\rm tr}}\left(\frac{e\mathcal{E}_\omega v_F}{\omega}\right)^2\frac{\omega_c^2+\omega^2}{(\omega^2-\omega_c^2)^2}, \qquad (10a)$$

$$\mathcal{Q}_{\rm dc}=\frac{2\,\tau_{\rm in}}{\tau_{\rm tr}}\left(\frac{e\mathcal{E}_{\rm dc}v_F}{\omega_c}\right)^2\left(\frac{\pi}{\omega_c}\right)^2. \qquad (10b)$$

With these notations, Eq. (9) reads

$$\begin{aligned}&\frac{\mathcal{P}_\omega}{4}\sum_\pm\tilde\nu(\varepsilon\pm\omega)\left[f(\varepsilon\pm\omega)-f(\varepsilon)\right]\\ &+\frac{\mathcal{Q}_{\rm dc}\,\omega_c^2}{4\pi^2\tilde\nu(\varepsilon)}\,\partial_\varepsilon\left[\,\tilde\nu^2(\varepsilon)\,\partial_\varepsilon f(\varepsilon)\,\right]=f(\varepsilon)-f_T(\varepsilon)\,. \qquad (11)\end{aligned}$$

Note that $\mathcal{P}_\omega$ and $\mathcal{Q}_{\rm dc}$ are proportional to $\tau_{\rm in}$ and are infinite in the absence of the inelastic relaxation processes.

The left-hand side of the kinetic equation (11), as well as Eqs. (1),(3) for the photoconductivity, can also be extracted from the quantum Boltzmann equation of Ref. 11, as we show in Appendix. Below, we use Eqs. (1),(3),(11) to analyze the photoconductivity in the both limiting cases of overlapping and separated LLs.

III. NON-EQUILIBRIUM DISTRIBUTION FUNCTION INDUCED BY MICROWAVE RADIATION

We consider first the case of overlapping LLs, with the DOS given by

$$\tilde{\nu} = 1 - 2\delta \cos\frac{2\pi\varepsilon}{\omega_c}, \quad \delta = \exp\left(-\frac{\pi}{\omega_c \tau_{\rm q}}\right) \ll 1. \qquad (12)$$

Here $\tau_{\rm q}$ is the zero-B single-particle relaxation time, which is much shorter than the transport time in high-mobility structures, $\tau_{\rm q} \ll \tau_{\rm tr}$ (because of the smooth character of a random potential of remote donors). The existence of a small parameter δ simplifies solution of the kinetic equation (11). To first order in δ, we look for a solution in the form

$$f = f_0 + f_{\rm osc} + O(\delta^2), \qquad f_{\rm osc} \equiv \delta\,{\rm Re}\left[f_1(\varepsilon)\, e^{i\frac{2\pi\varepsilon}{\omega_c}}\right]. \quad (13)$$

We assume that the temperature (measured in energy units, $k_{\rm B}=1$) is sufficiently high, $T \gg \omega_c$, implying a scale separation between the smooth energy dependence of functions $f_{0,1}(\varepsilon)$ on a scale of the order of T and the fast oscillations with a period ω_c .[14] We also assume that the electric fields are not too strong [$\mathcal{P}_\omega(\omega/T)^2 \ll 1$ and $\mathcal{Q}_{\rm dc}(\omega_c/T)^2 \ll 1$], so that the smooth part $f_0(\varepsilon)$ is close to the Fermi distribution $f_T(\varepsilon)$ at a bath temperature T; otherwise, the temperature of the electron gas is further increased due to heating.[15] We obtain[16]

$$f_{\rm osc}(\varepsilon) = \delta\,\frac{\omega_c}{2\pi}\,\frac{\partial f_T}{\partial \varepsilon}\,\sin\frac{2\pi\varepsilon}{\omega_c}\,\frac{\mathcal{P}_\omega \frac{2\pi\omega}{\omega_c}\sin\frac{2\pi\omega}{\omega_c} + 4\mathcal{Q}_{\rm dc}}{1+\mathcal{P}_\omega \sin^2\frac{\pi\omega}{\omega_c} + \mathcal{Q}_{\rm dc}}. \qquad (14)$$

Thus, the oscillations of the DOS $\nu(\varepsilon)$ induce an oscillatory contribution $f_{\rm osc}(\varepsilon)$ to the distribution function, as illustrated in Fig. 1.

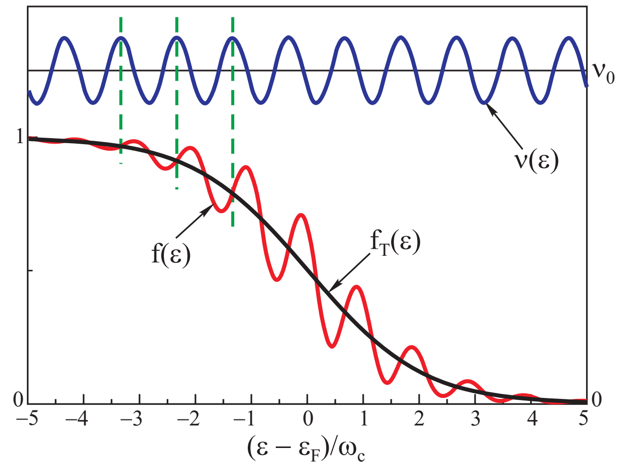

FIG. 1. Schematic behavior of the oscillatory density of states $\nu(\varepsilon)$ and radiation induced oscillations in the distribution function $f(\varepsilon)$ for $\sin(2\pi\omega/\omega_c) > 0$.

One might naively think that the small correction (14) to the distribution function will only weakly affect the conductivity. This is not the case, however. The reason is that, due to the fast oscillations in $f_{\rm osc}$, the derivative $\partial_\varepsilon f_{\rm osc}$ may be large. As a result, a small variation of the distribution function (14) can strongly affect the conductivity, Eq. (1). In particular, when the regions of an inverted population in $f(\varepsilon)$ correspond to the maxima in $\nu(\varepsilon)$ (as in Fig. 1), the linear–response conductivity may become negative, as we show below.

IV. OSCILLATORY PHOTOCONDUCTIVITY

To calculate the photoconductivity, we substitute Eq. (14) for the distribution function into Eq. (1). Performing the energy integration in Eq. (1), we assume (in conformity with the experiment) that T is much larger than the Dingle temperature, $T \gg 1/2\pi\tau_q$. Under this condition, the temperature smearing yields a dominant damping factor of the Shubnikov-de Haas oscillations, $X/\sinh X$ with $X = 2\pi^2 T/\omega_c$. The terms of order δ in Eq. (1) are then exponentially suppressed,

$$\delta \int d\varepsilon\, \cos\frac{2\pi\varepsilon}{\omega_c}\,\partial_\varepsilon f_T \propto \delta \exp(-2\pi^2 T/\omega_c) \ll \delta^2,$$

and can be neglected. The leading ω-dependent contribution to $\sigma_{\rm ph}$ comes from the δ^2 term generated by the product of $\partial_\varepsilon f_{\rm osc}(\varepsilon) \propto \delta\cos\frac{2\pi\varepsilon}{\omega_c}$ and the oscillatory part $-2\delta\cos\frac{2\pi\varepsilon}{\omega_c}$ of $\tilde{\nu}(\varepsilon)$. This term does survive the energy averaging, $-\int d\varepsilon\,\cos^2\frac{2\pi\varepsilon}{\omega_c}\,\partial_\varepsilon f_T \simeq 1/2$. We thus find

$$\frac{\sigma_{\rm ph}}{\sigma_{\rm dc}^{\rm D}} = 1 + 2\delta^2\left[1 - \frac{\mathcal{P}_\omega \frac{2\pi\omega}{\omega_c}\sin\frac{2\pi\omega}{\omega_c} + 4\mathcal{Q}_{\rm dc}}{1+\mathcal{P}_\omega \sin^2\frac{\pi\omega}{\omega_c} + \mathcal{Q}_{\rm dc}}\right]. \qquad (15)$$

Equation (15) is our central result. It describes the photoconductivity in the regime of overlapping LLs, including all non-linear (in $\mathcal{E}_\omega$ and $\mathcal{E}_{\rm dc}$) effects. Let us

analyze it in more detail. In the linear-response regime ($\mathcal{E}_{\rm dc} \to 0$) and for a not too strong microwave field, Eq. (15) yields a correction to the dark *dc* conductivity $\sigma_{\rm dc} = \sigma^{\rm D}_{\rm dc}(1+2\delta^2)$, which is linear in the microwave power:

$$\frac{\sigma_{\rm ph}-\sigma_{\rm dc}}{\sigma_{\rm dc}} = -4\delta^2\mathcal{P}_\omega\,\frac{\pi\omega}{\omega_c}\,\sin\frac{2\pi\omega}{\omega_c}, \tag{16}$$

in agreement with Ref. 12. It is enlightening to compare Eq. (16) with the contribution of the effect of the *ac* field on the impurity scattering.[9–11] The analytic result, Eq. (6.11) of Ref. 11, in the notation of Eq. (10) is

$$\frac{\sigma^{[11]}_{\rm ph}-\sigma_{\rm dc}}{\sigma_{\rm dc}} = -12\frac{\tau_{\rm q}}{\tau_{\rm in}}\delta^2\mathcal{P}_\omega\left(\frac{\pi\omega}{\omega_c}\,\sin\frac{2\pi\omega}{\omega_c}+\sin^2\frac{\pi\omega}{\omega_c}\right). \tag{17}$$

This result has a similar frequency dependence as Eq. (16); however, its amplitude is much smaller at $\tau_{\rm in} \gg \tau_{\rm q}$, i.e., the mechanism of Refs. 9–11 appears to be irrelevant. Physically, the effect of the *ac* field on the distribution function is dominant because it is accumulated during a diffusive process of duration $\tau_{\rm in}$, whereas Refs. 9–11 consider only one scattering event. Apart from the magnitude, the two contributions are qualitatively different in their temperature and polarization dependence. Specifically, the contribution related to the change of the distribution function is strongly temperature-dependent (due to the T-dependence of $\tau_{\rm in}$, see Sec. VII) and does not depend on the direction of the linear polarization of the microwave field. On the other hand, the effect of microwaves on the impurity collision integral yields a T-independent contribution which depends essentially on the relative orientation of the fields $\vec{\mathcal{E}}_\omega$ and $\vec{\mathcal{E}}_{\rm dc}$ [Eq. (17) represents the result averaged over the polarization direction].

With increasing microwave power, the photoconductivity saturates at the value

$$\frac{\sigma_{\rm ph}}{\sigma_{\rm dc}} = 1-8\delta^2\,\frac{\pi\omega}{\omega_c}\,\cot\frac{\pi\omega}{\omega_c}\;,\quad \mathcal{P}_\omega\sin^2\frac{\pi\omega}{\omega_c}\gg 1. \tag{18}$$

Note that although the correction is proportional to $\delta^2 \ll 1$, the factor $8\pi(\omega/\omega_c)\cot(\pi\omega/\omega_c)$ is large in the vicinity of the cyclotron resonance harmonics $\omega = k\omega_c$ ($k = 1, 2, \ldots$), and allows the photo-induced correction to exceed in magnitude the dark conductivity $\sigma_{\rm dc}$. In particular, $\sigma_{\rm ph}$ around minima becomes negative at $\mathcal{P}_\omega > \mathcal{P}^*_\omega > 0$, with the threshold value given according to Eq. (15) by

$$\mathcal{P}^*_\omega = \left(4\delta^2\frac{\pi\omega}{\omega_c}\sin\frac{2\pi\omega}{\omega_c}-\sin^2\frac{\pi\omega}{\omega_c}\right)^{-1}. \tag{19}$$

The evolution of the B dependence of the photoresistivity $\rho_{\rm ph}$ with increasing microwave power is illustrated in Fig. 2. More specifically, the curves in Fig. 2 correspond to different values of the dimensionless parameter

$$\mathcal{P}^{(0)}_\omega \equiv \mathcal{P}_\omega(\omega_c=0) = \frac{e^2}{\hbar c}\cdot\frac{\tau_{\rm in}}{\tau_{\rm tr}}\cdot\frac{v_F^2}{\omega^2 S}\cdot\frac{8\pi P}{\hbar\omega^2}, \tag{20}$$

where $P = c|\mathcal{E}_\omega|^2S/8\pi$ is the microwave power over the sample area S, c is the speed of light, and we restored the Planck constant for convenience.

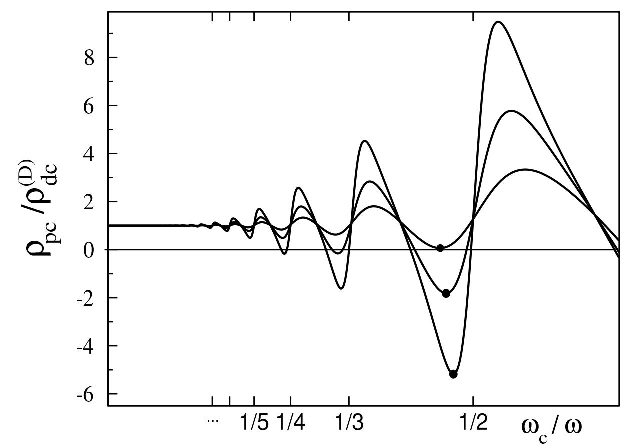

FIG. 2. Photoresistivity (normalized to the dark Drude value) for overlapping Landau levels vs ω_c/ω at fixed $\omega\tau_{\rm q} = 2\pi$. The curves correspond to different levels of microwave power $\mathcal{P}^{(0)}_\omega = \{0.24, 0.8, 2.4\}$. Nonlinear $I-V$ characteristics at the marked minima are shown in Fig. 3.

V. ZERO–RESISTANCE STATES

Let us now fix ω/ω_c such that $\mathcal{P}^*_\omega > 0$, and consider the dependence of $\sigma_{\rm ph}$ on the *dc* field $\mathcal{E}_{\rm dc}$ at a sufficiently strong microwave power $\mathcal{P}_\omega > \mathcal{P}^*_\omega$, corresponding to the negative linear–response photoconductivity. As follows from Eq. (15), in the limit of large $\mathcal{E}_{\rm dc}$ the conductivity is close to the Drude value and thus positive, $\sigma_{\rm ph} = (1-6\delta^2)\sigma^{\rm D}_{\rm dc} > 0$. Therefore, $\sigma_{\rm ph}$ changes sign at a certain value $\mathcal{E}^*_{\rm dc}$ of the *dc* field (see Fig. 3), which is determined by the condition $\mathcal{Q}_{\rm dc} = (\mathcal{P}_\omega - \mathcal{P}^*_\omega)/\mathcal{P}^*_\omega$. The negative-conductivity state at $\mathcal{E}_{\rm dc} < \mathcal{E}^*_{\rm dc}$ is unstable with respect to the formation of domains with a spontaneous electric field of the magnitude $\mathcal{E}^*_{\rm dc}$.[8] Using Eqs. (10), (19), we obtain

$$\begin{aligned}\mathcal{E}^*_{\rm dc} &= \frac{1}{\sqrt{2}\,\pi}\,\frac{\omega_c}{eR_c}\left(\frac{\tau_{\rm tr}}{\tau_{\rm in}}\right)^{1/2}\left[\left(\frac{\mathcal{E}_\omega}{\mathcal{E}^*_\omega}\right)^2-1\right]^{1/2}\\ &= \sqrt{\mathcal{E}^2_\omega-(\mathcal{E}^*_\omega)^2}\left[\frac{\omega_c^4(\omega^2+\omega_c^2)}{2\omega^2(\omega^2-\omega_c^2)^2}\right]^{1/2}\\ &\times\frac{1}{\pi}{\rm Re}\left(4\delta^2\frac{\pi\omega}{\omega_c}\sin\frac{2\pi\omega}{\omega_c}-\sin^2\frac{\pi\omega}{\omega_c}\right)^{1/2},\end{aligned} \tag{21}$$

with $\mathcal{E}^*_\omega$ being the threshold value of the *ac* field at which the ZRS develops and $R_c = v_F/\omega_c$ the cyclotron radius. Equation (21) relates the electric field formed in the domains (measurable by local probe[7]) with the excess power

of microwave radiation. It is worth noticing that the last expression for $\mathcal{E}^*_{\rm dc}$ does not explicitly contain the rate of the inelastic processes.

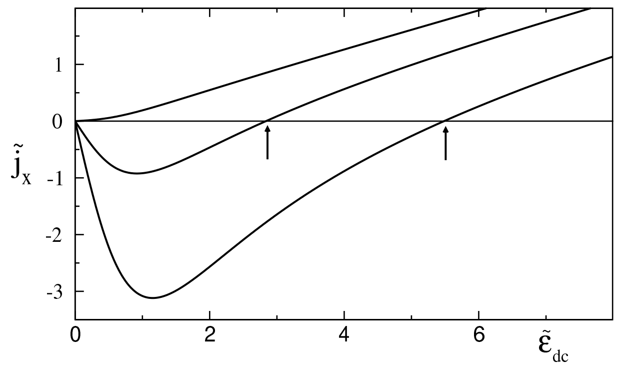

FIG. 3. Current–voltage characteristics [dimensionless current $\tilde{j}_x = (\sigma_{\rm ph}/\sigma^{\rm D}_{\rm dc})\tilde{\mathcal{E}}_{\rm dc}$ vs dimensionless field $\tilde{\mathcal{E}}_{\rm dc} = \mathcal{Q}^{1/2}_{\rm dc}$] at the points of minima marked by the circles in Fig. 1. The arrows show the *dc* field $\tilde{\mathcal{E}}^*_{\rm dc}$ in spontaneously formed domains.

VI. SEPARATED LANDAU LEVELS

We now turn to the regime of strong B, $\omega_c\tau_{\rm q}/\pi \gg 1$, where the LLs get separated. The DOS is then given (within the self-consistent Born approximation) by a sequence of semicircles of width $2\Gamma = 2(2\omega_c/\pi\tau_{\rm q})^{1/2}$:

$$\tilde{\nu}(\varepsilon) = \frac{2\omega_c}{\pi\Gamma^2}\sum_n \mathrm{Re}\sqrt{\Gamma^2 - (\varepsilon - n\omega_c - \omega_c/2)^2}. \qquad (22)$$

We use Eqs. (1) and (11) to evaluate the OPC at $\mathcal{Q}_{dc} \to 0$ to first order in $\mathcal{P}_\omega$ and estimate the correction of the second order. The condition $T \gg \omega_c$ allows us to separate the slow dependence on ε on the scale of T and fast oscillations with the period ω_c in the integral (1) by averaging over the period of the oscillations. After integrating the resulting slow-varying functions we obtain

$$\begin{aligned}\frac{\sigma_{\rm ph}}{\sigma^{\rm D}_{\rm dc}} &= \langle\, \tilde{\nu}^2(\varepsilon)\,\rangle_\varepsilon \\ &- \frac{\omega\mathcal{P}_\omega}{4}\,\langle\, [\,\tilde{\nu}(\varepsilon+\omega) - \tilde{\nu}(\varepsilon-\omega)\,]\,\partial_\varepsilon\tilde{\nu}^2(\varepsilon)\,\rangle_\varepsilon \\ &+ \omega\mathcal{P}^2_\omega \textstyle\sum_{k,l} a_{k,\,l}\,\langle\, \tilde{\nu}(\varepsilon+k\omega)\,\tilde{\nu}(\varepsilon+l\omega)\,\partial_\varepsilon\tilde{\nu}^2(\varepsilon)\,\rangle_\varepsilon \\ &+ O(\,\mathcal{P}^3_\omega\,). \qquad (23)\end{aligned}$$

Here the angular brackets denote averaging over ε within the period ω_c, and $a_{k,\,l}$ are numerical coefficients. The result for separated LLs, Eq. (22), reads

$$\begin{aligned}\frac{\sigma_{\rm ph}}{\sigma^{\rm D}_{\rm dc}} &= \frac{16\omega_c}{3\pi^2\Gamma}\left\{1 - \mathcal{P}_\omega\frac{\omega\omega_c}{\Gamma^2}\right. \\ &\times \left.\left[\sum_n \Phi\left(\frac{\omega - n\omega_c}{\Gamma}\right) + O\left(\frac{\omega_c\mathcal{P}_\omega}{\Gamma}\right)\right]\right\}, \qquad (24)\end{aligned}$$

$$\Phi(x) = \frac{3x}{4\pi}\mathrm{Re}\left[\arccos(|x|-1) - \frac{1+|x|}{3}\sqrt{|x|(2-|x|)}\right].$$

The photoresistivity for the case of separated LLs, Eq. (24), is shown in Fig. 4 for several values $\mathcal{P}_\omega$ of the microwave power. Notice that a correction to Eq. (24) of second order in $\mathcal{P}_\omega$ is still small even at microwave power exceeding the threshold value

$$\mathcal{P}^*_\omega \sim \Gamma^2/\omega\omega_c, \qquad (25)$$

since $\omega_c\mathcal{P}^*_\omega/\Gamma \sim \Gamma/\omega \ll 1$. This means that it suffices to keep the linear-in-$\mathcal{P}_\omega$ term only even for the microwave power $\mathcal{P}_\omega > \mathcal{P}^*_\omega$ at which the linear-response resistance becomes negative.

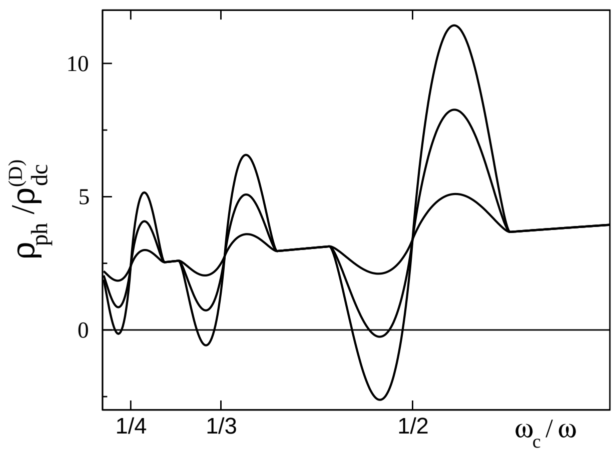

FIG. 4. Photoresistivity (normalized to the Drude value) for separated Landau levels vs ω_c/ω at fixed $\omega\tau_{\rm q} = 16\pi$. The curves correspond to different levels of microwave power $\mathcal{P}^{(0)}_\omega = \{0.01,\ 0.03,\ 0.05\}$.

As in the case of overlapping LLs, a negative value of the linear-response conductivity signals an instability leading to the formation of domains with the field $\mathcal{E}^*_{\rm dc}$ at which $\sigma_{\rm ph}(\mathcal{E}_{\rm dc}) = 0$. It turns out, however, that for separated LLs the kinetic equation in the form of Eq. (9) yields zero (rather than expected positive) conductivity in the limit of strong $\mathcal{E}_{\rm dc}$.[17] This happens because elastic impurity scattering between LLs, inclined in a strong *dc* field, is not included in Eq. (9). The inter-LL transitions become efficient in *dc* fields as strong as [see Eq. (5.5) of Ref. 11]

$$\mathcal{E}^*_{\rm dc} \sim \left(\frac{\tau_{\rm tr}}{\tau_{\rm q}}\right)^{1/2}\frac{\omega_c}{eR_c}, \qquad (26)$$

which actually gives the strength of the field in domains for the case of separated LLs.

VII. INELASTIC RELAXATION DUE TO ELECTRON–ELECTRON COLLISIONS

Finally, we calculate the inelastic relaxation time $\tau_{\rm in}$. Of particular importance is its T dependence which in

turn determines that of $\sigma_{\rm ph}$. At not too high T, the dominant mechanism of inelastic scattering is due to electron-electron (e-e) collisions. It is worth emphasizing that the e-e scattering does not yield relaxation of the total energy of the 2DEG and as such cannot establish a steady-state *dc* photoconductivity. That is to say the smearing of $f_0(\varepsilon)$ in Eq. (13), which is a measure of the degree of heating, is governed by electron-phonon scattering. However, the e-e scattering at $T \gg \omega_c$ does lead to relaxation of the oscillatory term $f_{\rm osc}$, Eq. (14), and thus determines the T behavior of the oscillatory contribution to $\sigma_{\rm ph}$.

Quantitatively, the effect of electron-electron interaction is taken into account by replacing the right-hand side of Eq. (9) by $-{\rm St}_{ee}\{f\}$, where the collision integral ${\rm St}_{ee}\{f\}$ is given by

$$\begin{aligned}{\rm St}_{ee}\{f\} =& \int d\varepsilon' \int dE\, A(E,\,\varepsilon,\,\varepsilon')\\ &\times\Big[-f(\varepsilon)f^{(h)}(\varepsilon_+)f(\varepsilon')f^{(h)}(\varepsilon'_-)\\ &\quad +f^{(h)}(\varepsilon)f(\varepsilon_+)f^{(h)}(\varepsilon')f(\varepsilon'_-)\Big], \quad (27)\end{aligned}$$

and $f^{(h)}(\varepsilon) \equiv 1 - f(\varepsilon)$, $\varepsilon_+ = \varepsilon + E$, $\varepsilon'_- = \varepsilon' - E$. The function $A(E,\,\varepsilon,\,\varepsilon')$ describes the dependence of the matrix element of the screened Coulomb interaction on the transferred energy E and the energies of colliding particles. In what follows we use the linearized form of ${\rm St}_{ee}$.

A. Overlapping Landau levels

For overlapping LLs, we put $\tilde\nu = 1$ in accord with the accuracy of Eq. (15). Then the kernel in Eq. (27) depends on the transferred energy E only and is given by the general formula[18]

$$A(E) = \frac{2\nu_0}{\pi}\int\frac{d^2q}{(2\pi)^2}\,|U|^2\,({\rm Re}\,\langle\mathcal{D}\rangle)^2\,. \quad (28)$$

Here the factor of two accounts for spin, $\langle\mathcal{D}\rangle(E,q)$ is the angle-averaged particle–hole propagator,

$$U(E,q) = \frac{\kappa/2\nu_0}{q + \kappa(1 + iE\langle\mathcal{D}\rangle)} \quad (29)$$

the dynamically screened Coulomb potential, and $\kappa = 4\pi e^2\nu_0$ the inverse screening length.

The propagator $\langle\mathcal{D}\rangle(E,q)$ has the (disorder-independent) Fermi-liquid form for ultra–ballistic momenta, $q \gg q_1 = (\omega_c/D)^{1/2}$, where $D = R_c^2/2\tau_{\rm tr}$ is the diffusion coefficient in a classically strong magnetic field,

$$\langle\mathcal{D}\rangle = (\,q^2v_F^2 - E^2\,)^{-1/2}\,, \quad q \gg q_1 = (\omega_c/D)^{1/2}\,. \quad (30)$$

With lowering q it crosses over into the quasiclassical particle–hole propagator in a smooth random potential ("ballistic diffuson")[19,20]

$$\langle\mathcal{D}\rangle = \sum_{n=0}^{\infty}\frac{J_n^2(qR_c)}{-i(E - n\omega_c) + Dq^2 + n^2/\tau_{\rm tr}}, \quad q \ll q_1\,, \quad (31)$$

where $J_n(x)$ is the Bessel function.

Let us calculate the kernel $A(E)$ for $E \lesssim T$ within the (experimentally relevant) temperature range,

$$\omega_c \ll T \ll \omega_c(\omega_c\tau_{\rm tr})^{1/2}. \quad (32)$$

The integration domain in Eq. (28) naturally divides into two parts: $q > q_1$ and $q < q_1$. In view of Eq.(32) the propagator (30) for ultra–ballistic momenta $q \gg q_1$ does not depend on the transferred energy $E \lesssim T$; specifically, $\langle\mathcal{D}\rangle \simeq 1/qv_F$, while the screening is effectively static, $U(E,q) \simeq U(0,q)$. It follows that the contribution $A^>(E)$ of $q \gtrsim q_1$ to the integral (28) has the form

$$A^>(E) = \frac{1}{2\pi\varepsilon_F}\ln\frac{\kappa}{q_1}. \quad (33)$$

By contrast, the contribution of $q \lesssim q_1$ to Eq. (28) is strongly E-dependent and is found as a sum over peaks coming from different n in Eq. (31),

$$A^<(E) = \sum_n A_n^<(E). \quad (34)$$

When calculating the contribution of a single peak $A_n^<$, the following approximations are justified: in Eq. (31), the product of the Bessel functions $J_n^2(qR_c)$ for the relevant momenta $q \gg R_c^{-1}$ can be replaced by $2\cos^2\varphi_n/\pi qR_c$ with $\varphi_n = qR_c + \pi(2n+1)/4$; the term $n^2/\tau_{\rm tr}$ in Eq. (31) can be neglected at $E \lesssim T$ in view of Eq. (32); also, the first term in the denominator of Eq. (29) can be omitted for $q \lesssim q_1 \ll \kappa$. Introducing the dimensionless parameters $\Delta_n(E) = |E/\omega_c - n| \le 1/2$, $\beta_n = 2\pi^2\omega_c\tau_{\rm tr}/n^2 \gg 1$, and $\gamma_n(E) = \beta_n^{1/3}\Delta_n(E)$ we express the contribution of the nth peak for $\Delta_n \ll 1$ as[21]

$$\begin{aligned}A_n^<(E) =& \frac{\beta_n^{2/3}}{4\pi^3\varepsilon_F}\int_0^\infty {\rm d}x\frac{x^5}{x^4 + \gamma_n^2}\\ &\times\left[\frac{x^6}{4\cos^4(\alpha_nx)} + \left(1 - \frac{\gamma_nx}{2\cos^2(\alpha_nx)}\right)^2\right]^{-1}. \quad (35)\end{aligned}$$

After averaging over the fast oscillations with α_nx, where $\alpha_n = n\beta_n^{1/3}/\pi$, the integration yields

$$A_n^<(E) = \frac{1}{4\pi^2\varepsilon_F}\begin{cases} c\,\beta_n^{2/3}, & \Delta_n \ll \beta_n^{-1/3} \\ 3/8\pi\Delta_n^2, & 1 \gg \Delta_n \gg \beta_n^{-1/3}\end{cases}, \quad (36)$$

where $c = \Gamma(7/6)/2^{1/3}(3\pi)^{1/2}\Gamma(2/3) \simeq 0.18$. We now use Eqs. (27), (33)–(36) to calculate the relaxation rate $\tau_{\rm in}^{-1}$ of the oscillatory component of the distribution function, $f_{\rm osc}(\varepsilon)$. As we are going to show, it is dominated by the large–momentum transfers, $q \gg q_1$. The contribution of this large–momentum region is easily evaluated: since

$A^>(E)$ is energy-independent, it is sufficient to take into account the out-scattering term only. We thus return to the right-hand side of Eq. (9) with the inelastic relaxation rate $\tau_{\rm in}^{-1}$ replaced by[22]

$$\tau_{ee}^{-1} = \int\limits_{-\infty}^{\infty} dE\, A^>(E)\, \frac{E}{2} \left(\coth\frac{E}{2T} - \tanh\frac{E+\varepsilon}{2T}\right) = \frac{\pi^2T^2+\varepsilon^2}{4\pi\varepsilon_F} \ln \frac{\kappa v_F}{\omega_c(\omega_c\tau_{\rm tr})^{1/2}}\ . \qquad (37)$$

Note that τ_{ee}^{-1} in Eq. (37) depends on B through the logarithmic factor only and crosses over into the conventional zero-B result $\tau_{ee}^{-1} = (\pi T^2/4\varepsilon_F)\ln(\kappa v_F/T)$ when T exceeds $\omega_c(\omega_c\tau_{\rm tr})^{1/2}$.

Let us now turn to the contribution of the region $q \lesssim q_1$. Evaluating the out-scattering term [similar to the first line of Eq. (37), with $A^>(E)$ replaced by $A^<(E)$, Eqs. (34),(36)], we find $(\tau_{\rm ee}^<)^{-1} \sim (\omega_c\tau_{\rm tr})^{1/3}\, T^2/\varepsilon_F$, which can exceed the large–q contribution (37). However, the relaxation time approximation is no longer valid for $q < q_1$. Indeed, the main contribution to $(\tau_{\rm ee}^<)^{-1}$ comes from the energy transfers E close to $n\,\omega_c$, $\Delta_n(E) \sim \beta_n^{-1/3}$, see Eq. (34). Such processes are inefficient as far as the relaxation of $f_{\rm osc}$ is concerned, since the energy transfer is almost commensurate with the period of the oscillations in the distribution function. In other words, the out–scattering term is almost compensated by the in-scattering one, so that $A^<(E) = \sum_n A_n^<(E)$ is effectively replaced by $\sum_n \Delta_n^2 A_n^<(E)$. As a result, the contribution of region of $q \lesssim q_1$ to the relaxation rate is dominated by $q \sim q_1$ and is given by Eq. (37) without the logarithmic factor, and thus can be neglected.

In fact, the situation is similar to the momentum relaxation due to small-angle scattering off a smooth random potential. The momentum relaxation time $\tau_{\rm tr}$ and the out-scattering (or single-particle) relaxation time $\tau_{\rm q}$ in that case differ by the factor $(1-\cos\phi)$, which accounts for a reduced contribution to the resistivity of small-angle scattering with $\phi \ll 1$:

$$\left.\begin{array}{l}\tau_{\rm q}^{-1}\\ \tau_{\rm tr}^{-1}\end{array}\right\} = 2\pi\nu_0 \int \frac{d\phi}{2\pi}\, W(2k_F\sin\frac{\phi}{2}) \times \left\{\begin{array}{l}1\\ (1-\cos\phi)\end{array}\right.\ ,$$

where $W(q)$ is the Fourier transform of the correlation function of the random potential. A similar result for the relaxation of the oscillatory part of the distribution functions is obtained from Eq. (27). We linearize Eq. (27) with the distribution function in the form (we assume $T \gg \omega$)

$$f = f_T + \varphi(\varepsilon)\, \partial_\varepsilon f_T\ , \qquad (38)$$

where $\varphi(\varepsilon) = \varphi(\varepsilon+\omega_c)$. This ansatz is suggested by Eq. (14) and will be confirmed by the calculation below. We substitute Eq. (38) in Eq. (27) and use the condition $T \gg \omega_c$ which allows us to separate the slow dependence on E, ε' on the scale of T and fast oscillations with the period ω_c by averaging over the period of the oscillations. Using $A(E) = A(E+\omega_c)$ [see Eqs. (33), (34), and (36)], we obtain the following integrals over the slow variables which all produce the same result,

$$\begin{aligned}
&-\int d\varepsilon' \int dE\, [f_T^{(h)}(\varepsilon_+) f_T(\varepsilon') f_T^{(h)}(\varepsilon'_-) \\
&\qquad + f_T(\varepsilon_+) f_T^{(h)}(\varepsilon') f_T(\varepsilon'_-)]\, \partial_\varepsilon f_T(\varepsilon) \\
&= \int d\varepsilon' \int dE\, [f_T(\varepsilon) f_T(\varepsilon') f_T^{(h)}(\varepsilon'_-) \\
&\qquad + f_T^{(h)}(\varepsilon) f_T^{(h)}(\varepsilon') f_T(\varepsilon'_-)]\, \partial_\varepsilon f_T(\varepsilon_+) \\
&= -\int d\varepsilon' \int dE\, [f_T(\varepsilon) f_T^{(h)}(\varepsilon_+) f_T^{(h)}(\varepsilon'_-) \\
&\qquad + f_T^{(h)}(\varepsilon) f_T(\varepsilon_+) f_T(\varepsilon'_-)]\, \partial_{\varepsilon'} f_T(\varepsilon') \\
&= \int d\varepsilon' \int dE\, [f_T(\varepsilon) f_T^{(h)}(\varepsilon_+) f_T(\varepsilon') \\
&\qquad + f_T^{(h)}(\varepsilon) f_T(\varepsilon_+) f_T^{(h)}(\varepsilon')]\, \partial_{\varepsilon'} f_T(\varepsilon'_-) \\
&= \frac{\pi^2T^2+\varepsilon^2}{2} \partial_\varepsilon f_T(\varepsilon)\,. \qquad (39)
\end{aligned}$$

The collision integral thus reads

$$-{\rm St}_{\rm in}\{f\} = \frac{\pi^2T^2+\varepsilon^2}{2}\, \frac{\partial f_T}{\partial\varepsilon} \times \langle\, A(E)\,[\,\varphi(\varepsilon) - \varphi(\varepsilon+E) + \varphi(\varepsilon') - \varphi(\varepsilon'-E)\,]\,\rangle_{\varepsilon',\,E}\ , \qquad (40)$$

where the angular brackets denote averaging over ε' and E within the period ω_c. For a harmonic modulation of the distribution function, $\varphi(\varepsilon) \propto \cos(2\pi\varepsilon/\omega_c + \theta)$, as in Eq. (13), and using $A(E) = A(-E)$, we obtain

$$-{\rm St}_{\rm in}\{f\} = \frac{f - f_T}{\tau_{\rm in}}\ ,$$
$$\tau_{\rm in}^{-1} = \frac{\pi^2T^2+\varepsilon^2}{2}\, \langle\, A(E)\,[\,1 - \cos(2\pi E/\omega_c)\,]\,\rangle_E\ . \qquad (41)$$

Because of the factor $1-\cos(2\pi E/\omega_c)$ the contribution of small momenta (34), (36) to the relaxation rate is small compared to that of $q \gtrsim q_1$. In the latter case, due to the energy-independent kernel $A(E)$, Eq. (33), the in-scattering part is zero on average, and $\tau_{\rm in}^{-1}$ coincides with the out-scattering rate (37).

The inelastic relaxation time $\tau_{\rm in}$ as obtained above [Eqs. (37),(41)] depends on energy ε.[23] This makes the problem somewhat more complicated than the model considered in Sec. II with a phenomenological, ε–independent parameter $\tau_{\rm in}$. However, characteristic energies are $\varepsilon \sim T$ [see Eq. (38)], so that the ε–dependence in Eqs. (37),(41) does not change the T^{-2} scaling of $\tau_{\rm in}$ but only yields a numerical factor. In particular, repeating the analysis of Eq. (9) in the linear–in–$\mathcal{P}_\omega$ regime with the ε–dependent $\tau_{\rm in}$, we find that $\tau_{\rm in}$ entering Eq. (10a) is effectively replaced by

$$\int d\varepsilon\, \tau_{\rm in}(\varepsilon,T)\,(-\partial_\varepsilon f_T) = \tau_{\rm in}(0,T) \int d\varepsilon \frac{-\partial_\varepsilon f_T}{1+(\varepsilon/\pi T)^2} \simeq 0.822\, \tau_{\rm in}(0,T). \qquad (42)$$

Therefore, at $T \gg \omega$ the linear photoresistivity, Eq. (16), scales as T^{-2}.

At $T \ll \omega$ (but still $T \gg \omega_c$), the ε–dependence in $\tau_{\rm in}$ becomes more important. Indeed, solving Eq. (9) to the linear order in $\mathcal{E}_\omega^2$ (and at $\mathcal{E}_{\rm dc} = 0$), we find the following oscillatory contribution to the distribution function

$$f_{\rm osc}(\varepsilon) = \tau_{\rm in}(\varepsilon, T)\, \mathcal{E}_\omega^2 \frac{\sigma_\omega^{\rm D}}{2\omega^2\nu_0} \times \sum_{\pm} \tilde\nu(\varepsilon \pm \omega)\, [\, f_T(\varepsilon \pm \omega) - f_T(\varepsilon)\,]. \qquad (43)$$

Equation (1) then reproduces Eq. (16) for the photoconductivity, with $\tau_{\rm in}$ (entering $\mathcal{P}_\omega$) given by

$$\tau_{\rm in} = \int \frac{d\varepsilon}{2\omega} \tau_{\rm in}(\varepsilon, T)[f_T(\varepsilon - \omega) - f_T(\varepsilon + \omega)]. \qquad (44)$$

For $T \gg \omega$ this expression reduces back to Eq. (42), while in the opposite limit $T \ll \omega$ it yields

$$\tau_{\rm in} = \int \frac{d\varepsilon}{2\omega} \tau_{\rm in}(\varepsilon, T) = \frac{\pi^2 T}{2\omega}\, \tau_{\rm in}(0, T). \qquad (45)$$

Thus, the T^{-2} scaling of the photoresistivity at $T \gg \omega$ transforms into the T^{-1} behavior for $T \ll \omega$.

B. Separated Landau levels

We turn now to the case of separated LLs. Let us assume again that T is not too high,

$$\omega_c \ll T \ll \Gamma(\tau_{\rm tr}/\tau_{\rm q})^{1/2}\ . \qquad (46)$$

The dominant contribution, similarly to the case of overlapping LLs, is given by large transferred momenta $q \gg (\Gamma/D_{\rm B})^{1/2}$, where $D_{\rm B} = R_c^2/2\tau_{\rm tr,\,B}$ is the diffusion coefficient in a quantizing magnetic field (see Sec. II). However, the situation for a strongly oscillating DOS, Eq. (22), is different in that it is no longer sufficient to deal solely with the out-scattering processes even for large q; that is the whole collision integral (27) should be taken into account. The kernel in Eq. (27) may be re-written as:

$$A(E,\ \varepsilon,\ \varepsilon') = \frac{2}{\pi^3\nu_0\tilde\nu(\varepsilon)} \int \frac{d^2q}{(2\pi)^2}\ |U|^2\, \Pi^q_{\varepsilon,\,\varepsilon+E}\, \Pi^q_{\varepsilon',\,\varepsilon'-E}\, , \qquad (47)$$

where the function Π for large $q \gg (\Gamma/D_{\rm B})^{1/2}$ reads[24]

$$\Pi^q_{\varepsilon,\varepsilon'} = \pi\nu_0 (q v_F)^{-1} \tilde\nu(\varepsilon)\tilde\nu(\varepsilon')\, . \qquad (48)$$

The procedure leading to Eq. (40) is applicable in the case of separated LLs as well. As a result, the inelastic collision term on the right-hand side of Eq. (9) should be replaced by

$$-{\rm St}_{\rm in}\{f\} = \frac{\pi^2 T^2 + \varepsilon^2}{4\pi\varepsilon_F \tilde\nu(\varepsilon)}\ \ln \frac{\kappa v_F \tau_{\rm q}^{1/2}}{\omega_c \tau_{\rm tr}^{1/2}}\ \frac{\partial f_T}{\partial \varepsilon} \times \langle\, \tilde\nu(\varepsilon)\tilde\nu(\varepsilon+E)\tilde\nu(\varepsilon')\tilde\nu(\varepsilon'-E) \times [\, \varphi(\varepsilon) - \varphi(\varepsilon+E) + \varphi(\varepsilon') - \varphi(\varepsilon'-E)\,]\, \rangle_{\varepsilon',E}\ . \qquad (49)$$

In other words, Eq. (9) becomes an integral equation for the periodic function $\varphi(\varepsilon)$ characterizing oscillations of the distribution function,

$$\mathcal{A}\, [\, \tilde\nu(\varepsilon+\omega) - \tilde\nu(\varepsilon-\omega)\,] = \langle\, \tilde\nu(\varepsilon+E)\, \tilde\nu(\varepsilon')\, \tilde\nu(\varepsilon'-E) \times [\, \varphi(\varepsilon) - \varphi(\varepsilon+E) + \varphi(\varepsilon') - \varphi(\varepsilon'-E)\,]\, \rangle_{\varepsilon',E}\ , \qquad (50)$$

where $\mathcal{A}$ is a smooth function of ε,

$$\mathcal{A} = \frac{2\pi\varepsilon_F \mathcal{E}_\omega^2\, \sigma_\omega^{\rm D}}{\omega\nu_0(\pi^2T^2 + \varepsilon^2)}\ \ln^{-1} \frac{\kappa v_F \tau_{\rm q}^{1/2}}{\omega_c \tau_{\rm tr}^{1/2}}\, .$$

Analytical solution of Eq. (50) does not seem feasible. However, up to a factor of order unity,[25] we can rewrite the exact collision integral (49) in the relaxation-time approximation, thus returning to Eq. (9) with

$$\tau_{\rm in}^{-1} \sim \frac{\omega_c}{\Gamma}\ \frac{T^2 + (\varepsilon/\pi)^2}{\varepsilon_F}\ \ln \frac{\kappa v_F \tau_{\rm q}^{1/2}}{\omega_c \tau_{\rm tr}^{1/2}}\ . \qquad (51)$$

One sees that the T and ε dependence of $\tau_{\rm in}$ in the regime of separated LLs is the same as for overlapping LLs [Eqs. (37),(41)]. Therefore the temperature scaling of the linear-in-$\mathcal{P}_\omega$ photoresistivity, Eq. (24), is the same as found in Sec. VII A. In particular, $\sigma_{\rm ph} - \sigma_{\rm dc}$ scales as T^{-2} for $T \gg \omega$.

VIII. OSCILLATORY HALL RESISTIVITY

Finally, let us briefly discuss the issue of the microwave induced ω/ω_c–oscillations of the Hall resistivity ρ^H_{xy} (antisymmetric part of ρ_{xy}, i.e. $\rho^H_{xy} = -\rho^H_{yx}$) detected in recent experiments.[26] The experimentally observed oscillations of ρ_{xy} demonstrate the following properties. First, they have the same period and the opposite phase as compared to the oscillations of the dissipative resistivity ρ_{xx}. Second, the amplitude of the oscillations in ρ_{xy} is roughly the same as in ρ_{xx}. Third, the radiation-induced contribution $\delta\rho_{xy}$ is odd with respect to the magnetic field B, which implies that the measured $\delta\rho_{xy}$ is indeed a contribution to the Hall resistivity ρ^H_{xy}. These observations cannot be explained within either the mechanism related to the effect of the microwaves on the distribution function (Ref. 12 and this work) or the one related to the effect on the elastic collision integral (Refs. 9–11), if one assumes, as usual, that the electron density n_e is constant. If n_e was constant, the leading odd-in-B correction $\delta\sigma^H_{yx}$ to the Hall conductivity $\sigma^H_{yx} = cen_{\rm e}/B$ should be smaller than $\delta\sigma_{xx}$ by a factor $1/\omega_c\tau_{\rm tr}$, which is of order 10^{-2} under the experimental conditions. Therefore, the

observed $\delta\rho_{xy}^H$, although very small in comparison with the bare ρ_{xy}, appears to be two order of magnitude larger than what one would expect from the theory of the oscillatory contribution to ρ_{xy} neglecting the oscillations of n_e.[27] A possible origin of the observed oscillatory ρ_{xy} may be a weak variation of n_e with ω/ω_c. The observation[28] of a variation of the period of the Shubnikov-de Haas oscillations, $\delta\sigma_{xx} \propto \cos(c\,n_e/eB)$, appears to support the idea of the microwave-induced oscillations of the electron density. The issue warrants further study.

IX. CONCLUSION

To summarize, we have presented a theory of magnetooscillations in the photoconductivity of a 2DEG. The parametrically largest contribution to the effect is governed by the microwave-induced change in the distribution function. We have analyzed the nonlinearity with respect to both the microwave and *dc* fields. The result takes an especially simple form in the regime of overlapping LLs, Eq. (15). We have shown that the magnitude of the effect is governed by the inelastic relaxation time (44), (41), (51), and increases as T^{-2} or T^{-1} (depending on the relation between T and ω) with lowering temperature. For a sufficiently strong microwave power the linear-in-*dc*-field photoconductivity becomes negative leading to formation of domains with zero resistivity. We have calculated the threshold power at which this zero–resistance state is formed, Eq. (19), and the spontaneous *dc* field in the domains, Eq. (21).

Our results are in overall agreement with the experimental findings.[2,3,30] The observed T dependence of the photoresistivity at maxima compares well with the predicted T^{-2} behavior. Typical parameters $\omega/2\pi \simeq 50-100$ GHz, $\tau_\mathrm{q} \simeq 10$ ps yield $\omega\tau_\mathrm{q}/2\pi \simeq 0.5-1$ (overlapping LLs), and the experimental data indeed closely resemble Fig. 2. For $T \sim 1$ K and $\epsilon_F \sim 100$ K we find $\tau_\mathrm{in}^{-1} \sim 10$ mK, much less than $\tau_\mathrm{q}^{-1} \sim 1$ K, as assumed in our theory. For the microwave power $P \sim 1$ mW and the sample area $S \sim 1\,\mathrm{cm}^2$, Eq. (20) yields the dimensionless power $\mathcal{P}_\omega^{(0)} \sim 0.005-0.1$ (the smaller value corresponds to $\omega/2\pi = 100$ GHz, the larger one to $\omega/2\pi = 50$ GHz), where we used $\tau_\mathrm{tr} = 10$ mK and $v_F = 2\cdot 10^7$ cm/s. These values of $\mathcal{P}_\omega^{(0)}$ agree with characteristic values for separated LLs (Fig. 4) but are noticeably less than the prediction for overlapping LLs (Fig. 2). The discrepancy can be attributed (at least partly) to the fact that the value of τ_q used in the above estimate, which was extracted from the Shubnikov–de Haas experiments, is in fact masked by inhomogeneous broadening and thus is shorter than the actual value. Indeed, τ_q found from the (experimental) damping of the oscillations in ρ_ph (which are local and thus not affected by inhomogeneous broadening) according to Eq. (15) is several times longer. With this value of τ_q the threshold microwave power $\mathcal{P}_\omega^*$, Eq. (19), needed for the emergence of the zero-resistance states, corresponds to P less than 1 mW, in conformity with the experiments.[33] Finally, for $\mathcal{P}_\omega - \mathcal{P}_\omega^* \sim \mathcal{P}_\omega^*$, at $T \sim 1$ K (when $\tau_\mathrm{tr}/\tau_\mathrm{in} \sim 1$), and $\omega_c/2\pi = 50$ GHz the estimated *dc* electric field in the domains, Eq. (21), is found to be $\mathcal{E}_\mathrm{dc}^* \sim 1$ V/cm. This is in agreement with the experimental data of Ref. 7 where the voltage drop between an internal and an external contact (separated by 200 μm) generated by the radiation in the absence of the drive current was of the order of 5 mV for $\omega_c/2\pi \simeq 20$ GHz. Assuming that the size of the domain is of the order of the system size, this yields $\mathcal{E}_\mathrm{dc}^* \sim 0.25$ V/cm. One sees that this value indeed compares well with our theoretical estimate $\mathcal{E}_\mathrm{dc}^* \sim 1$ V/cm, especially taking into account the ω_c^2 dependence of $\mathcal{E}_\mathrm{dc}^*$ following from Eq. (21).

Recently, a number of publications appeared that extended our theory (main results of which were presented in Ref. 13) in a variety of contexts: propagation of surface-acoustic waves,[34,35] photoconductivity of laterally-modulated structures,[36] photoconductivity for B above the cyclotron resonance,[37] local compressibility of irradiated samples.[38]

We thank R.R. Du, K. von Klitzing, R.G. Mani, J.H. Smet, and M.A. Zudov for information about the experiments, and I.V. Gornyi for numerous stimulating discussions. This work was supported by the SPP "Quanten-Hall-Systeme" of the DFG, by NSF grants DMR02-37296, DMR 02-13282 and EIA02-10376, by AFOSR grant F49620-01-1-0457, and by the RFBR.

APPENDIX: DERIVATION OF THE BASIC EQUATIONS FROM THE QUANTUM BOLTZMANN EQUATION

The purpose of this Appendix is to demonstrate how the semiclassical transport theory of Ref. 11 (see Sec. III of this reference) is reduced to Eqs. (1),(3),(11) when the effect of electric fields on the impurity collision process can be neglected and only the distribution function is affected. The conditions under which the effect of the fields on the collision integral is weak are that the *dc* field is much smaller than E_0 and the strength of the *ac* field satisfies $\mathcal{P} \ll 1$, where E_0 and $\mathcal{P}$ are defined in Eqs. (5.5) and (6.2) of Ref. 11, respectively. In notation of our Eq. (10) these conditions read[29]

$$\mathcal{P}_\omega,\ \mathcal{Q}_\mathrm{dc} \ll \tau_\mathrm{in}/\tau_\mathrm{q}\ . \tag{A1}$$

Under the conditions (A1) we neglect the effect of the external field on the density of states, which amounts to putting $h_1 = 1$ in Eq. (3.42) of Ref. 11. The resulting DOS is time independent:

$$\nu(\varepsilon) = \nu_0 \left[1 + 2\mathrm{Re}\sum_{l=1}^{\infty} \lambda^l g_l \exp\left(\frac{i2\pi l\varepsilon}{\omega_c}\right)\right], \tag{A2}$$

where the coherence factor $\lambda = -\exp(-\pi/\omega_c\tau_q)$,

$$g_l = l^{-1}\, L_{l-1}^1(2\pi l/\omega_c\tau_\mathrm{q}),$$

and L_l^m is the Laguerre polynomial. The combination $|g_l\lambda^l|^2$ has the meaning of the probability for an electron to complete the cyclotron orbit after l revolutions. Here we follow the notations of Ref. 11; in the main text of the present paper the factor λ is denoted as $-\delta$.

The kinetic equations (3.46) of Ref. 11 are written in the time representation for the distribution function $f(t,t';\varphi,\mathbf{R})$, where ϕ is the angle on the cyclotron orbit and $\mathbf{R}$ is the position of its guiding center. This function is related to the conventional distribution in the energy-time representation by the Wigner transform

$$f(t,t') = \int \frac{d\varepsilon}{2\pi} e^{-i\varepsilon(t-t')} f\left(\frac{t+t'}{2},\varepsilon\right). \tag{A3}$$

The inverse Wigner transform of both sides of Eq. (3.46a) of Ref. 11 yields the canonical form of the Boltzmann equation for the distribution function

$$(\partial_t + \omega_c\partial_\varphi) f\,(t,\varepsilon;\varphi,\mathbf{R}) = \mathrm{St}_{\mathrm{im}}\{f\}_{\varepsilon,t} + \mathrm{St}_{\mathrm{in}}\{f\}_{\varepsilon,t}. \tag{A4}$$

A time independent version $\mathrm{St}_{\mathrm{in}}\{f\}_{\varepsilon,t}$ of the inelastic term is considered in Sec. VII and will not be discussed further in this Appendix. The impurity collision term $\mathrm{St}_{\mathrm{im}}\{f\}_{\varepsilon,t}$ is obtained as the inverse Wigner transform of Eq.(3.46b) of Ref. 11 and will be written explicitly below.

Under the condition (A1), the dissipative electric current at the point $\mathbf{r}$ is obtained from Eq. (A3) and Eq. (3.45b) of Ref. 11:

$$\mathbf{j}^{(d)}(\mathbf{r},t) = 2ep_F \int_0^{2\pi} \frac{d\varphi}{2\pi}\mathbf{i}(\phi) \int \frac{d\varepsilon}{2\pi} \Bigg\{ f\,(t,\varepsilon;\varphi,\mathbf{r}_g)$$
$$+2\mathrm{Re}\sum_{l=1}^{\infty} \lambda^l g_l \exp\left(\frac{i2\pi\varepsilon}{\omega_c}\right) f\left(t-\frac{\pi l}{\omega_c},\varepsilon;\varphi,\mathbf{r}_g\right)\Bigg\}, \tag{A5}$$

where p_F is the Fermi momentum, $\mathbf{r}_g = \mathbf{r} - R_c\,\hat\epsilon\,\mathbf{i}(\varphi) - \boldsymbol{\zeta}(t)$ is the guiding center coordinate and $\boldsymbol{\zeta}(t)$ describes the motion of an electron in the external field $\mathbf{E}(t)$:

$$\partial_t\boldsymbol{\zeta}(t) = \left(\frac{\partial_t - \omega_c\hat\epsilon}{\partial_t^2+\omega_c^2}\right)\frac{e\mathbf{E}(t)}{m}, \tag{A6}$$

$\mathbf{i} = \{\cos\varphi, \sin\varphi\}$, $\hat\epsilon$ is the antisymmetric tensor, $\epsilon_{xy} = -\epsilon_{yx} = 1$.

In view of Eq. (A1) one can neglect the effect of the electric fields on the interference processes in the collision integral (3.46b) of Ref. 11, which amounts to putting the form-factors $h_1 = h_2 = 1$ in Eq. (3.49). Then Eq. (3.46b) in the time representation acquires a simple form

$$\mathrm{St}_{\mathrm{im}}\{f\}_{t,t'} = -\frac{\hat L(t,t')}{\tau_{\mathrm{tr}}} f(t,t') \tag{A7}$$
$$+\frac{1}{\tau_{\mathrm{tr}}}\sum_{l=1}^{\infty}\lambda^l g_l \left\{\left[M_l(t) - \hat L\,(t,t')\right] f\left(t-\frac{2\pi l}{\omega_c},t'\right)\right\}$$
$$+\frac{1}{\tau_{\mathrm{tr}}}\sum_{l=1}^{\infty}\lambda^l g_l \left\{\left[M_l(t') - \hat L\,(t,t')\right] f\left(t,t'-\frac{2\pi l}{\omega_c}\right)\right\},$$

where

$$\hat L(t,t') = \left(\,\{\,p_F[\,\boldsymbol{\zeta}(t) - \boldsymbol{\zeta}(t')\,]\hat\epsilon - iR_c\nabla_R\,\}\cdot\mathbf{i}(\varphi) + i\partial_\varphi\,\right)^2$$

and $M_l(t)$ is the result of action of the operator $\hat L\,(t,\,t-2\pi l/\omega_c)$ on unity. Performing the Wigner transformation of Eq. (A7) we obtain

$$\mathrm{St}_{\mathrm{im}}\{f\}_{\varepsilon,t} =$$
$$-\frac{1}{\tau_{\mathrm{tr}}}\hat L\left(t+\frac{i}{2}\partial_\varepsilon\,,\,t-\frac{i}{2}\partial_\varepsilon\right) f(t,\varepsilon;\varphi,\mathbf{R}) + \frac{2}{\tau_{\mathrm{tr}}}\mathrm{Re}\sum_{l=1}^{\infty}\lambda^l g_l$$
$$\times\left\{\left[M_l\left(t+\frac{i}{2}\partial_\varepsilon\right) - \hat L\left(t+\frac{i}{2}\partial_\varepsilon\,,\,t-\frac{i}{2}\partial_\varepsilon\right)\right]\right.$$
$$\left.\times\exp\left(\frac{2i\pi l\varepsilon}{\omega_c}\right) f\left(t-\frac{\pi l}{\omega_c},\varepsilon;\varphi,\mathbf{R}\right)\right\}. \tag{A8}$$

Equations (A4) and (A8) are valid for an arbitrary time dependent distribution function. We now notice from Eq. (A4) that the most divergent terms in the distribution function are due to the contribution of the angular and time independent part of f to the collision integral. For $\omega_c\tau_{\mathrm{tr}} \gg 1$ it is sufficient to consider f to be time, angular, and coordinate independent. Equation (A8) reduces to

$$\overline{\mathrm{St}_{\mathrm{im}}\{f\}_{\varepsilon,t}} = -\frac{N\,(i\partial_\varepsilon)}{\tau_{\mathrm{tr}}} f(\varepsilon) + \frac{2}{\tau_{\mathrm{tr}}}\mathrm{Re}\sum_{l=1}^{\infty}\lambda^l g_l$$
$$\times\left\{\left[N\left(\frac{2\pi l}{\omega_c}\right) - N\,(i\partial_\varepsilon)\right]\exp\left(\frac{2i\pi l\varepsilon}{\omega_c}\right) f\,(\varepsilon)\right\},$$
$$N\,(\delta t) \equiv \frac{p_F^2}{2}\overline{[\,\boldsymbol{\zeta}(t) - \boldsymbol{\zeta}\,(t-\delta t)\,]^2}. \tag{A9}$$

and the bar stands for the time averaging. In the constant electric field $N\,(\delta t) = (\delta t)^2 p_F^2(\partial_t\boldsymbol{\zeta})^2/2$, whereas for the microwave field $\boldsymbol{\zeta}(t) = \mathrm{Re}\,(\boldsymbol{\zeta}_\omega e^{i\omega t})$ and $N\,(\delta t) = p_F^2\mathrm{Re}\,[\,\boldsymbol{\zeta}_\omega\boldsymbol{\zeta}_\omega^*(1-e^{i\omega\delta t})\,]/2$. Substituting these expressions in Eq. (A9), taking $\boldsymbol{\zeta}$ from Eq. (A6), and using Eq. (A2), we arrive at the left-hand side of Eq. (9).

The zero angular harmonic of the distribution function does not contribute to the electric current (A5) directly. The relevant angular dependent correction $\delta f(\varphi)$, with $\int d\varphi\,\delta f(\varphi) = 0$, can be found perturbatively from Eqs. (A4) and (A8):

$$\omega_c\partial_\varphi\delta f\,(\varphi) = p_F\tau_{\mathrm{tr}}^{-1}\partial_\varphi\left[\overline{\partial_t\boldsymbol{\zeta}(t)}\,\hat\epsilon\,\mathbf{i}(\varphi)\right]$$
$$\times\left[1 + 2\mathrm{Re}\sum_{l=1}^{\infty}\lambda^l g_l\exp\left(\frac{2\pi il\varepsilon}{\omega_c}\right)\right]\partial_\varepsilon f(\varepsilon). \tag{A10}$$

Substituting the solution of Eq. (A10) in Eq. (A5) and applying Eq. (A6) in the limit of the dc field, we obtain Eq. (1) with $\sigma_{\mathrm{dc}}(\varepsilon)$ given by Eq. (3).

[*] Also at A.F. Ioffe Physico-Technical Institute, 194021 St. Petersburg, Russia.

[†] Also at Petersburg Nuclear Physics Institute, 188350 St. Petersburg, Russia.

[1] M.A. Zudov, R.R. Du, J.A. Simmons, and J.L. Reno, Phys. Rev. B **64**, 201311(R) (2001); P.D. Ye, L.W. Engel, D.C. Tsui, J.A. Simmons, J.R. Wendt, G.A. Vawter, and J.L. Reno, Appl. Phys. Lett. **79**, 2193 (2001).

[2] R.G. Mani, J.H. Smet, K. von Klitzing, V. Narayanamurti, W.B. Johnson, and V. Umansky, Nature **420**, 646 (2002).

[3] M.A. Zudov, R.R. Du, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. **90**, 046807 (2003).

[4] R.G. Mani, J.H. Smet, K. von Klitzing, V. Narayanamurti, W.B. Johnson, and V. Umansky, Phys. Rev. B **70**, 155310 (2004); Phys. Rev B **69**, 193304 (2004); Phys. Rev. Lett. **92**, 146801 (2004).

[5] C.L. Yang, M.A. Zudov, T.A. Knuuttila, R.R. Du, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. **91**, 096803 (2003).

[6] S.I. Dorozhkin, JETP Lett. **77**, 577 (2003).

[7] R.L. Willett, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. **93**, 026804 (2004).

[8] A.V. Andreev, I.L. Aleiner, and A.J. Millis, Phys. Rev. Lett. **91**, 056803 (2003).

[9] A.C. Durst, S. Sachdev, N. Read, and S.M. Girvin, Phys. Rev. Lett. **91**, 086803 (2003).

[10] V.I. Ryzhii, Sov. Phys. Solid State **11**, 2078 (1970); V.I. Ryzhii, R.A.Suris, and B.S. Shchamkhalova, Sov. Phys. Semicond. **20**, 1299 (1986).

[11] M.G. Vavilov and I.L. Aleiner, Phys. Rev. B **69**, 035303 (2004).

[12] I.A. Dmitriev, A.D. Mirlin, and D.G. Polyakov, Phys. Rev. Lett. **91**, 226802 (2003).

[13] I.A. Dmitriev, M.G. Vavilov, I.L. Aleiner, A.D. Mirlin, and D.G. Polyakov, cond-mat/0310668; Physica E **25**, 205 (2004).

[14] As a matter of fact, the analytic results work numerically well down to $T \sim \omega_c$. The reason is that the actual small parameter is $\omega_c/2\pi T \ll 1$. In particular, this is illustrated in Fig. 1, where $T = \omega_c$ but the scale separation between the fast oscillations and the smooth energy dependence of $f_T(\varepsilon)$ is evident. Related to that is the temperature damping of the Shubnikov-de Haas oscillations $\exp(-2\pi^2 T/\omega_c)$.

[15] In fact, due to the heating the temperature T may itself become an oscillatory function of the ratio ω/ω_c, inducing an additional contribution to the OPC. In particular, the quasiclassical memory effects lead to such oscillations of T even in the absence of DOS oscillations, see I.A. Dmitriev, A.D. Mirlin, and D.G. Polyakov, Phys. Rev. B **70**, 165305 (2004). The corresponding contribution to the OPC is, however, small compared to the effect considered in the present paper.

[16] Strictly speaking, Eq.(14) is written under the assumption $T \gg \omega$ which is stronger than the condition $T \gg \omega_c$ for high oscillation harmonics, $\omega/\omega_c \gg 1$. This condition, however, only helps to simplify technically the solution of Eq.(11) but is not essential for the physics, which is based on the separation of energy scales (fast oscillations vs. smooth dependence) and requires $T \gg \omega_c$ only. In particular, it is not difficult to check (see Sec. VII) that the linear-in-microwave-power result (16) remains valid independently of the relation between T and ω.

[17] One can show that for an arbitrary $\nu(\varepsilon)$, Eq. (11) yields for $Q_{\rm dc} \gg 1$ the conductivity $\sigma = \sigma_{\rm dc}^{\rm D}/\langle\tilde\nu^{-2}(\varepsilon)\rangle_\varepsilon$, which is positive for overlapping LLs and vanishes in the presence of spectral gaps.

[18] B.L. Altshuler and A.G. Aronov, in *Electron-Electron Interactions in Disordered Systems*, edited by A.L. Efros and M. Pollak (North-Holland, Amsterdam, 1985).

[19] A.M. Rudin, I.L. Aleiner, and L.I. Glazman, Phys. Rev. B **58**, 15698 (1998).

[20] I.V. Gornyi and A.D. Mirlin, Phys. Rev. B **69**, 045313 (2004).

[21] For $\Delta_n \sim 1$, $A_n^<(E)$ is governed by momenta $q \sim q_1$, so that $A_n^<(E) \sim 1/\varepsilon_F$.

[22] The energy ε is counted in Sec. VII from the Fermi level.

[23] Note that despite the energy dependence of $\tau_{\rm in}$ on the scale of $\varepsilon \sim T$, the collision integral in (41) conserves the number of particles in view of fast oscillations of $f - f_T$ with a period $\omega_c \ll T$.

[24] A.V. Khaetskii and Y.V. Nazarov, Phys. Rev. B **59**, 7551 (1999).

[25] Strictly speaking, this factor depends on the argument $(\omega - n\omega_c)/\Gamma$ of the function Φ in Eq. (24), which describes the shape of oscillations in Fig. 4. Therefore, the precise shape of oscillations will be somewhat modified if one uses the exact collision integral (49) instead of the relaxation time approximation (9). However, all the qualitative features of Fig. 4 will remain intact.

[26] S.A. Studenikin, M. Potemski, P.T. Coleridge, A. Sachrajda, Z.R. Wasilewski, Solid State Comm. **129**, 341 (2004); R.G. Mani, V. Narayanamurti, K. von Klitzing, J.H. Smet, W.B. Johnson, and V. Umansky, Phys. Rev. B **69**, 161306 (2004).

[27] An attempt to explain the oscillations of the Hall conductivity was made by V.I. Ryzhii, J. Phys. Soc. Japan **73**, 1539 (2004) and Physica E **24**, 96 (2004). However, we believe that the claims made in these works that their results explain the experiments[26] are incorrect. Indeed, what the author calculates is actually the xy component of the anisotropic dissipative conductivity, $\Delta\sigma_{xy} = \Delta\sigma_{yx}$, related to the effect of microwaves on the collision integral.[11] This component is even in B, $\Delta\sigma_{xy}(B) = \Delta\sigma_{xy}(-B)$, and thus is not relevant to the Hall conductivity, $\Delta\sigma_{xy}^H(B) = -\Delta\sigma_{xy}^H(-B)$.

[28] R. Mani, private communication (2004).

[29] Throughout the paper we assume that $\tau_{\rm in}/\tau_{\rm q} \gg 1$. Under this condition, the neglected effect of the electric fields on the collision integral is much weaker than the effect on the distribution function, retained in Eq. (9).

[30] It was claimed in recent papers[31,32] that the effect of microwaves on the collision integral studied in Refs. 9–11 can explain the experimental data. We disagree with these claims. First, this effect is small compared to the one studied in the present paper by a factor $\tau_q/\tau_{\rm in}$ (see Sec. IV), which is of order 10^{-2} at $T \sim 1$K, and is thus much smaller than what is experimentally observed (unfortunately, the results of Refs. 31, 32 are presented in the form of proportionality relations, which makes a comparison of the magnitude of the effect to the experiment impossible). An agree-

ment with the experimental data claimed in Ref. 31 may be due to the fact that the authors used for their estimate the microwave power provided by a Gunn diode,[3] while the attenuated power incident on the sample was smaller by more than two orders of magnitude. Second, the contribution related to the collision integral is T-independent and polarization-dependent (as discussed in Sec. IV), in contrast to the experiment. An attempt[32] to explain the temperature dependence by a T-dependent LL width determined by the electron-electron interaction is in contradiction with the fact that the inelastic scattering rate $\tau_{\rm ee}^{-1}$ is two orders of magnitude smaller than the impurity-scattering rate τ_q^{-1} at $T \sim 1\,$K.

[31] V. Ryzhii and R. Suris, J. Phys. Cond. Matter **15**, 6855 (2003).

[32] V. Ryzhii, A. Chaplik, and R. Suris, JETP Letters **80**, 363 (2004).

[33] Interpolating between Figs. 2 and 4, we estimate the dimensionless microwave power necessary for the zero-resistance state according to our theory as $\mathcal{P}_\omega^{(0)} \sim 0.05$. This value agrees with the dimensionless power necessary for 50 GHz ($\mathcal{P}_\omega^{(0)} = 0.1$), which we estimate using the experimental data [according to Eq. (20)] and assuming that there are no standing waves in the cavity. There remains, however, some discrepancy with the value of power (0.005) for 100 GHz, which might possibly be attributed to the fact that both the power and the distribution of the electromagnetic field in the cavity are not very well known in the current experiments.

[34] J.P. Robinson, M.P. Kennett, N.R. Cooper, and V.I. Fal'ko, Phys. Rev. Lett. **93**, 036804 (2004).

[35] J.P. Robinson, M.P. Kennett, N.R. Cooper, and V.I. Fal'ko, cond-mat/0410064.

[36] J. Dietel, L. Glazman, F. Hekking, and F. von Oppen, Phys. Rev. B **71**, 045329 (2005).

[37] S.I. Dorozhkin, J.H. Smet, V. Umansky, and K. von Klitzing, cond-mat/0409228.

[38] M.G. Vavilov, I.A. Dmitriev, I.L. Aleiner, A.D. Mirlin, and D.G. Polyakov, Phys. Rev. B **70**, 161306(R) (2004).